Privacy for Data: Masking Methods

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V. Torra (2022) A guide to data privacy, Springer (Chapter 3)

Outline

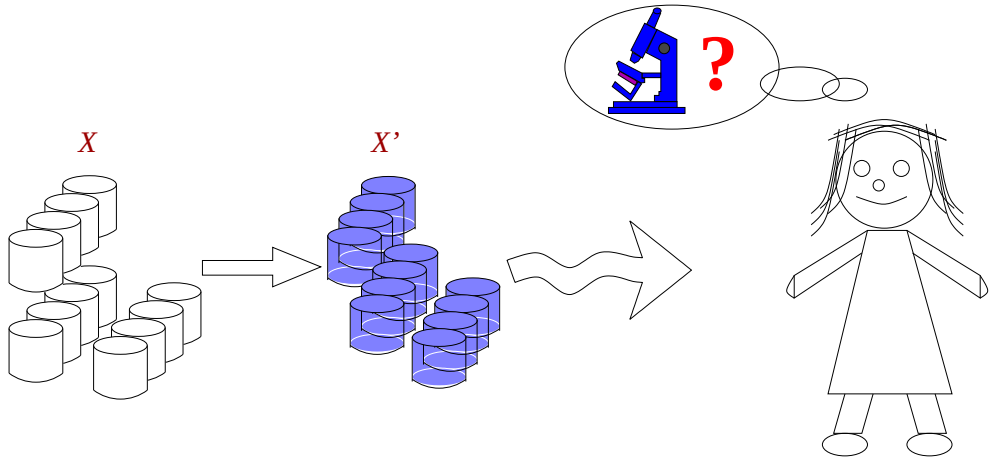
Introduction

Data-driven protection procedures

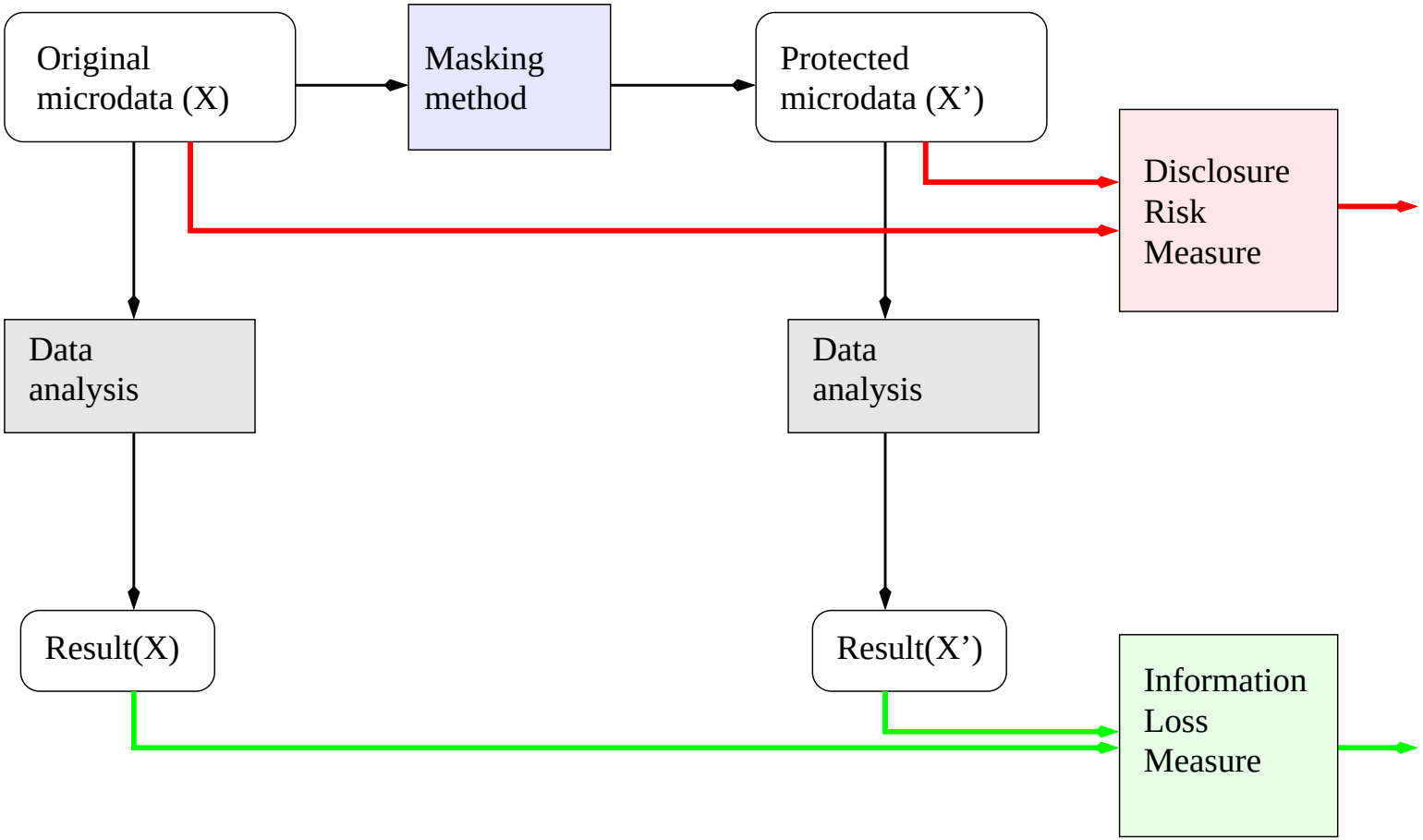
Data-driven protection methods

Data protection methods

- Data files / Microdata
- Aggregate data / Tabular data
- Other types of data
  - graphs for online social networks
  - search and access logs
  - documents or index of documents



Data-driven protection methods



Protection for microdata (files)

Data-driven protection methods

Data protection methods for datafiles / microdata

- Perturbative methods
- Non-perturbative methods
- Synthetic data generators

Data-driven protection methods

Data protection methods for datafiles / microdata

- **Perturbative.** The original data set is distorted in some way, and the new data set might contain some erroneous information.

Data-driven protection methods

Data protection methods for datafiles / microdata

- **Perturbative.** The original data set is distorted in some way, and the new data set might contain some erroneous information.
  For example, noise is added to an attribute following a $N(0, a)$ for a given a.
  Some combinations of values disappear, and, new combinations appear in the protected data set.
  At the same time, combinations in the protected data set no longer correspond to the ones in the original data set. This obfuscation makes disclosure difficult for intruders.

Data-driven protection methods

Data protection methods for datafiles / microdata

- **Non-perturbative.** Protection is achieved through replacing an original value by another one that is not incorrect but less specific.

Data-driven protection methods

Data protection methods for datafiles / microdata

- **Non-perturbative.** Protection is achieved through replacing an original value by another one that is not incorrect but less specific. For example, we replace a real number by an interval.
  In general, non-perturbative methods reduce the level of detail of the data set. This detail reduction causes different records to have the same combinations of values, which makes disclosure difficult to intruders.

Data-driven protection methods

Data protection methods for datafiles / microdata

- **Synthetic Data Generators.** In this case, instead of distorting the original data, new artificial data is generated and used to substitute the original values.
  Formally, synthetic data generators build a data model from the original data set and, subsequently, a new (protected) data set is randomly generated constrained by the model computed.

Data-driven protection methods

Data types

- Numerical data
- Categorical data: ordinal and nominal scale
  - Ordinal: $<$ (elements can be ordered)
  - No order predefined
- Logs
  - search and access logs
- Longitudinal data and time series
- Smart grid
- Mobility and location data
- Graphs and social networks
- Documents
- Image and video

Data-driven protection methods

Data protection methods for datafiles / microdata

- Perturbative methods
  - Noise addition, Microaggregation, Rank Swapping, ...
- Non-perturbative methods
  - Suppression, generalization, top and bottom coding, ...
  - Some versions of microaggregation
- Synthetic data generators
  - GANs, IPSO, ...

Perturbative methods

Rank Swapping

Rank swapping

- Description with parameter p
  - Values are ordered in increasing order
    We assume them ordered $x_{ij} \leq x_{lj}$ for all $1 \leq i < l \leq n$
  - Each ranked value x_{ij} is swapped with another ranked value x_{lj} randomly chosen within a restricted range $i < l \leq i + p$
- In applications, each variable is masked independently
- The larger the p,
  the larger the information loss, and the lower the risk

Rank swapping

Example I: Protection

- Four variables with values $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- rank swapping with $p = 2$.

Original file				Protected file			
a_1	a_2	a_3	a_4	a'_1	a'_2	a'_3	a'_4
8	9	1	3	10	10	3	5
6	7	10	2	5	5	8	1
10	3	4	1	8	4	2	2
7	1	2	6	9	2	4	4
9	4	6	4	7	3	5	6
2	2	8	8	4	1	10	10
1	10	3	9	3	9	1	7
4	8	7	10	2	6	9	8
5	5	5	5	6	7	6	3
3	6	9	7	1	8	7	9

Rank swapping

- Properties
  - Properties of a individual variables are kept
    e.g., means and variances
  - Correlations are modified
    because variables are masked independently
    modification depends on the data and on the number of records

Rank swapping

Example II: Information Loss and Disclosure Risk

(Nin, Herranz, Torra, DKE)

- Information Loss: IL = 39.22
- Disclosure Risk:
  - DLD = 17.5
  - PLD = 0.0
  - ID = 44.81
  - DR = 0.25DLD + 0.25PLD + 0.5 ID = 26.78
- Score = (IL+DR)/2 = 33
- Each DR measure (DLD, PLD, ID) as an average of four scenarios:

	DBRL	PRL
$\{V_1\}$	0	0
$\{V_1, V_2\}$	2	0
$\{V_1, V_2, V_3\}$	4	0
$\{V_1, V_2, V_3, V_4\}$	1	0
Average	17.5	0

Rank swapping

Specific record linkage for rank swapping

- Rank swapping record linkage (RS-RL).
  - If we know p, a given intruder's (original) record can only generate at most $2p$ records

Rank swapping

Specific record linkage for rank swapping

- Rank swapping record linkage (RS-RL).
  - If we know p, a given intruder's (original) record can only generate at most $2p$ records
  - For each x_{ij} of the intruder,
  - there exists a computable set $B(x_{ij})$ of $2p$ masked records, that can be generated from the original record x_i

Rank swapping

Example III: Specific record linkage for rank swapping

- Record (intruder) $x_2 = (6, 7, 10, 2)$, $p = 2$ and first variable $x_{21} = 6$
  - $B(x_{21} = 6) = \{(4, 1, 10, 10), (5, 5, 8, 1), (6, 7, 6, 3), (7, 3, 5, 6), (8, 4, 2, 2)\}$

Original file				Protected file				$B(x_{2j})$
a_1	a_2	a_3	a_4	a'_1	a'_2	a'_3	a'_4	$B(x_{21})$
8	9	1	3	10	10	3	5	
6	7	10	2	5	5	8	1	X
10	3	4	1	8	4	2	2	X
7	1	2	6	9	2	4	4	
9	4	6	4	7	3	5	6	X
2	2	8	8	4	1	10	10	X
1	10	3	9	3	9	1	7	
4	8	7	10	2	6	9	8	
5	5	5	5	6	7	6	3	X
3	6	9	7	1	8	7	9	

Rank swapping

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Rank swapping

Specific record linkage for rank swapping

- Rank swapping record linkage (RS-RL).
  - If we know p, a given intruder's (original) record can only generate at most $2p$ records
  - For each x_{ij} of the intruder,
  - there exists a computable set $B(x_{ij})$ of $2p$ masked records, that can be generated from the original record x_i
- It should happen that the masked record is in all $B(x_{ij})$

$$x'_\ell \in \cap_{1 \leq j \leq c} B(x_{ij}).$$

Rank swapping

Example IV: Specific record linkage for rank swapping

- Record (intruder) $x_2 = (6, 7, 10, 2)$, $p = 2$ and 2nd var. $x_{22} = 7$
  - $B(x_{22} = 7) = \{(5, 5, 8, 1), (2, 6, 9, 8), (6, 7, 6, 3), (1, 8, 7, 9), (3, 9, 1, 7)\}$

Original file				Protected file				$B(x_{2j})$	
a_1	a_2	a_3	a_4	a'_1	a'_2	a'_3	a'_4	$B(x_{21})$	$B(x_{22})$
8	9	1	3	10	10	3	5		
6	7	10	2	5	5	8	1	X	X
10	3	4	1	8	4	2	2	X	
7	1	2	6	9	2	4	4		
9	4	6	4	7	3	5	6	X	
2	2	8	8	4	1	10	10	X	
1	10	3	9	3	9	1	7		X
4	8	7	10	2	6	9	8		X
5	5	5	5	6	7	6	3	X	X
3	6	9	7	1	8	7	9		X

Rank swapping

Example V: Specific record linkage for rank swapping

- Similarly:
  - $B(x_{21} = 6) = \{(4, 1, 10, 10), (5, 5, 8, 1), (6, 7, 6, 3), (7, 3, 5, 6), (8, 4, 2, 2)\}$
  - $B(x_{22} = 7) = \{(5, 5, 8, 1), (2, 6, 9, 8), (6, 7, 6, 3), (1, 8, 7, 9), (3, 9, 1, 7)\}$
  - $B(x_{23} = 10) = \{(5, 5, 8, 1), (2, 6, 9, 8), (4, 1, 10, 10)\}$
  - $B(x_{24} = 2) = \{(5, 5, 8, 1), (8, 4, 2, 2), (6, 7, 6, 3), (9, 2, 4, 4)\}$
- The intersection of these sets ...
  - is the single record $(5, 5, 8, 1)$.

    $\rightarrow$ this is the correct link

    When several records are present, we apply standard record linkage.

Rank swapping

Example VI: Specific record linkage for rank swapping

- Scores:
  - With previous record linkage algorithm:
    - ▷ DR = 0.25DLD + 0.25PLD + 0.5 ID = 26.78
    - ▷ Score = (IL+DR)/2 = 33
  - Using only RS-RL:
    - ▷ DR = 0.5RS-RL + 0.5 ID = 43.655
    - ▷ Score = (IL+DR)/2 = 41.44

	DBRL	PRL	RS-RL
$\{V_1\}$	0	0	0
$\{V_1, V_2\}$	2	0	2
$\{V_1, V_2, V_3\}$	4	0	7
$\{V_1, V_2, V_3, V_4\}$	1	0	8
Average	17.5	0	42.5

Rank swapping

Discussion

- Specific record linkage improve the results of generic record linkage
- The implementation detects cases where reidentification is achieved and detected (i.e., the intruder knows that reidentification has taken place)
  - $KR.Reid(K.Reid(B, A), Reid(B, A))$
    Know that identification takes place + prob. of reidentification
- The attack can be done with a single record
- The more attributes the intruder knows, the better
  intersection will have less records
- A lower bound of p can be found, if unknown

Microaggregation with individual ranking (univariate) can also be attacked effectively

Rank swapping

Transparency-aware rank swapping

- Development of masking methods resilient to specific record linkage methods:
  - rank swapping p-distribution
  - rank swapping p-buckets
- Their goal is to have $B_j = X$, so, $\cap_j B_j = X$.

Rank swapping

Transparency-aware rank swapping

- rank swapping p-buckets
  - Sort values and cluster in p buckets: $B_1, \ldots, B_p$
  - For each value a_i of bucket B_r select a value a_l of a bucket B_s. Select B_s with $s \geq r$ according

$$Pr[B_s ischoose | B_r] = \frac{1}{K} \frac{1}{2^{s-r+1}}.$$

Select a_l from bucket B_s using uniform distribution, use $l > i$ if $B_s = B_r$

Rank swapping

- Variants of swapping for other types of data
  E.g., location privacy: swap trajectories

- Sliding window for streaming data

Microaggregation

Microaggregation

- Informal definition. Small clusters are built for the data, and then each record is replaced by a representative.

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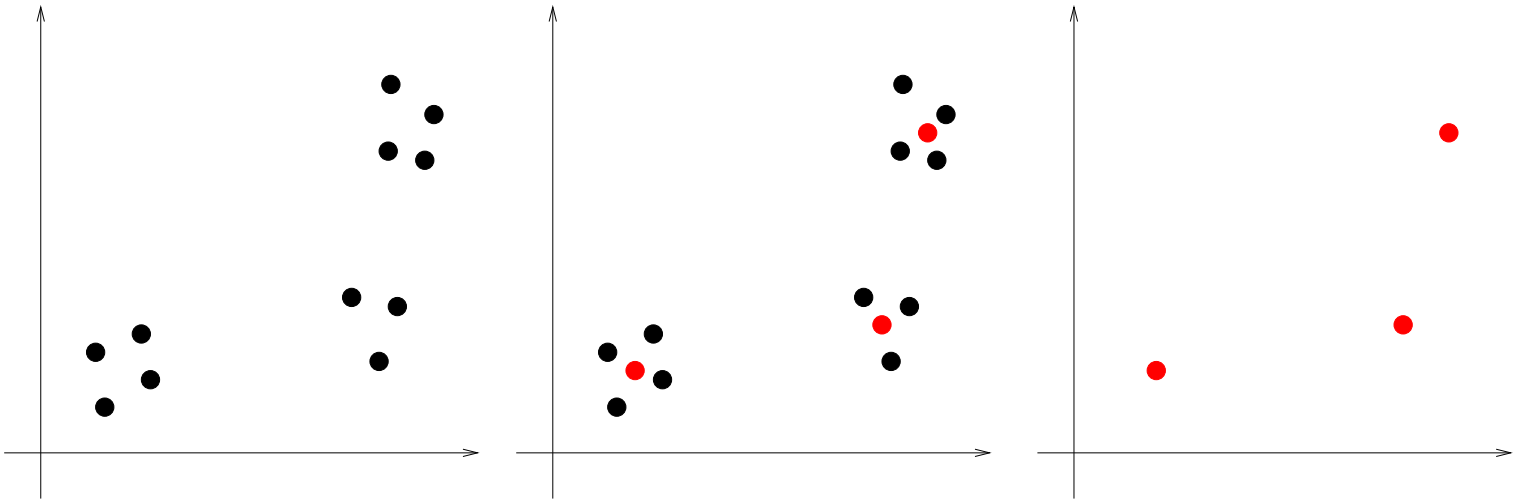
- Disclosure risk and information loss
  - Low disclosure is ensured requiring k records in each cluster
  - Low information loss is ensured as clusters are small

Microaggregation

- Operational definition. It is defined in terms of
  - **Partition.** Records are partitioned into several clusters, each of them consisting of at least k records.
  - **Aggregation.** For each of the clusters a representative (the centroid) is computed
  - **Replacement.** The original records are replaced by the representative of the cluster to which they belong to.

Microaggregation

- Graphical representation of the process.



Microaggregation

- Formalization. u_{ij} to describe the partition of the records in X. That is, $u_{ij} = 1$ if record j is assigned to the ith cluster. Let v_i be the representative of the ith cluster, then a general formulation of microaggregation with g clusters and a given k is as follows:

$$\begin{aligned}
\text{Minimize} \quad & SSE = \sum_{i=1}^{g} \sum_{j=1}^{n} u_{ij} (d(x_j, v_i))^2 \\
\text{Subject to} \quad & \sum_{i=1}^{g} u_{ij} = 1 \text{ for all } j = 1, \ldots, n \\
& 2k \geq \sum_{j=1}^{n} u_{ij} \geq k \text{ for all } i = 1, \ldots, g \\
& u_{ij} \in \{0, 1\}
\end{aligned}$$

Microaggregation

- Optimality
  - Polynomial solution when only one variable
  - Optimal solution is NP-hard for more than 2 variables
  - Heuristic methods have been developed:
    MDAV, Projected microaggregation

Microaggregation

- Heuristic approaches
  - usually follow the operational approach
    - ▷ **Build a partition.**
    - ▷ **Define an aggregation.** Mean of the records in the cluster
    - ▷ **Replacement.**

Microaggregation

- Multivariate case
  - When a file has several variables
    - Microaggregate all the variables at once
    - Microaggregate sets of variables
    - Microaggregate one variable at a time: individual ranking

Microaggregation: One variable/individual ranking

- **Algorithm** Optimal Univariate
  Data: $X = (a_1 \dots a_n)$: original data set (single attribute), k: integer
  Result: X': protected data set
  begin
    $A :=$ Sort the values of X in ascending order so that if $i < j$ then $a_i \leq a_j$.
    Given A and k, a graph $G_{k,n}$ is defined as follows.
    begin
      Define the nodes of G as the elements a_i in A plus one additional node g_0 (this node is later needed to apply the Dijkstra algorithm).
      For each node g_i, add to the graph the directed edges (g_i, g_j) for all j such that $i + k \leq j < i + 2k$. The edge (g_i, g_j) means that the values $(a_{i+1}, \dots, a_j)$ might define one of the possible clusters.
      The cost of the edge (g_i, g_j) is defined as the within-group sum of squared error for such cluster. That is, $SSE = \Sigma_{l=i+1}^{j}(a_l - \bar{a})^2$, where $\bar{a}$ is the average record of the cluster.
    end
    Compute the shortest path between the nodes g_0 and g_n. This shortest path can be computed using the Dijkstra algorithm. Each edge represents a cluster
    $X' :=$ replace each value in X by the average record of its corresponding cluster
    return X'
  end

Microaggregation

- **Algorithm** General Multivariate Microaggregation
  Data: X: original data set, k: integer

  Result: X': protected data set

  begin
  Let $\Pi = \{\pi_1, \ldots, \pi_p\}$ be a partition of the set of attributes $V = \{V_1, \ldots, V_s\}$
  foreach $\pi \in \Pi$ **do**
  Microaggregate X considering only the attributes in π
  end

  end

Microaggregation

- Heuristic algorithms for microaggregation
  - Projection in one dimensional space + optimal microaggregation
  - Adhoc algorithms
    - MDAV, spanning trees, other clustering-based algorithms

Microaggregation

- **Algorithm** Projected Microaggregation
  Data: X: original data set, k: integer

  Result: X': protected data set

  begin

  Apply a projection algorithm to X, and obtain an univariate vector $z = (z_1, \ldots, z_n)$ where n is the number of records and z_i the projection of the ith record
  Sort the components of z in increasing order
  Apply optimal univariate microaggregation to the vector z, using as cost between nodes the within-group sum of square error of the records associated to these nodes (i.e., the SSE of the full records and not only of the projections)
  For each cluster resulting from the previous step, compute the s-dimensional centroid and replace all the records in the cluster by the centroid

  end

Microaggregation

- **Algorithm** MDAV microaggregation

begin

$C = \emptyset$

while $|X| \geq 3k$ **do**

$\bar{x}$ = the average record of all records in X

x_r = the most distant record from $\bar{x}$

x_s = the most distant record from x_r

C_r = cluster around x_r (with x_r and the $k-1$ closest records to x_r)

C_s = cluster around x_s (with x_s and the $k-1$ closest records to x_s)

Remove records in C_r and C_s from data set X

$C = C \cup \{C_r, C_s\}$

end

if $|X| \geq 2k$ **then**

$\bar{x}$ = the average record of all records in X

x_r = the most distant record from $\bar{x}$

C_r = cluster around x_r (with x_r and the $k-1$ closest records to x_r)

$C_s = X \setminus C_r$ (form another cluster with the rest of records)

$C = C \cup \{C_r, C_s\}$

end

else

$C = C \cup \{X\}$

end

end

Microaggregation

- Discussion and summary (I)
  - The larger the k, the lower the risk, the larger the information loss
  - Microaggregation is related to k-anonymity:
    all variables microaggregated together imply k-anonymity
  - It is easy to define microaggregation for other types of data
    distance, and aggregation method (plurality rule - most frequent value)
    E.g, application to logs, sets of documents (via bags of words), graphs
    time series (different distances produce different effects)

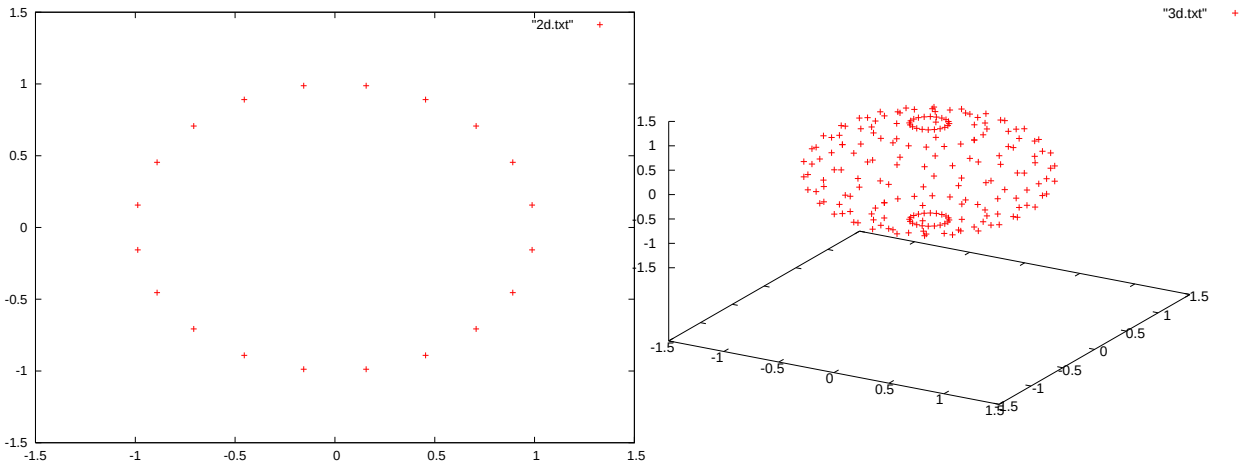
Microaggregation

- Discussion and summary (II)
  - Means kept, variance decreased
  - Correlation? It depends
  - Correlated variables together or not?

    Most usually correlated variables are microaggregated together to keep correlations in the protected data set.

    Microaggregation of two unrealistic datasets give worse results grouping correlated attributes than not grouping them.

    Not clear conclusion with real data.



Microaggregation

- Discussion and summary (III)
  - Post-masking for microaggregation (it is heuristic-based!)
    - ▷ Improve data quality (utility) after protection.
    - ▷ Reassign some records
    - ▷ Add a property updating X': Blow-up microaggregation

Microaggregation

- Discussion and summary (IV)
  - Transparency attacks to microaggregation
    - Parameters can be found even if not provided
    - Individual ranking attack effective: similar to rank swapping
  - Transparency-aware microaggregation
    - Fuzzy microaggregation

Additive noise

Additive noise

- Description:
  - This method protects data adding noise into the original file.

$$X' = X + \epsilon,$$

    where ϵ is the noise.
  - The simplest approach is to require ϵ such that $E(\epsilon) = 0$ and $Var(\epsilon) = kVar(X)$ for a given constant k.

Additive noise

- Description:
  - This method protects data adding noise into the original file.

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    where ϵ is the noise.
  - The simplest approach is to require ϵ such that $E(\epsilon) = 0$ and $Var(\epsilon) = kVar(X)$ for a given constant k.

- Properties:
  - No assumptions about the range of possible values for V_i (which may be infinite).
  - The noise added is typically continuous and with mean zero, which suits continuous original data well.
  - No exact matching is possible with external files.

Additive noise

- Uncorrelated noise
  - For variables V_i and V_j, noise is such that $Cov(\epsilon_i, \epsilon_j) = 0$ for $i \neq j$.
    - Uncorrelated additive noise preserves means and covariances.

$$E(X') = E(X) + E(\epsilon) = E(X)$$

$$Cov(X'_i, X'_j) = Cov(X_i, X_j) \text{ for } i \neq j$$

$$Var(X') = Var(X) + kVar(X) = (1+k)Var(X)$$

$$\begin{aligned} \rho_{X'_i,X'_j} &= \frac{Cov(X'_i, X'_j)}{\sqrt{Var(X'_i)Var(X'_j)}} = \frac{Cov(X_i, X_j)}{(1+k)\sqrt{Var(X_i)Var(X_j)}} \\ &= \frac{1}{1+k}\rho_{X_i,X_j} \end{aligned}$$

Additive noise

- Correlated noise
  - ϵ follows a normal distribution $N(0, k\Sigma)$ where Σ is the covariance matrix of X.
    - ▷ It preserves correlation coefficients and means.

$$E(X') = E(X) + E(\epsilon) = E(X)$$

$$Cov(X'_i, X'_j) = (1+k)Cov(X_i, X_j) \text{ for } i \neq j$$
$$Var(X') = Var(X) + kVar(X) = (1+k)Var(X)$$

$$\begin{aligned} \rho_{X'_i,X'_j} &= \frac{Cov(X'_i, X'_j)}{\sqrt{Var(X'_i)Var(X'_j)}} = \frac{(1+k)Cov(X_i, X_j)}{(1+k)\sqrt{Var(X_i)Var(X_j)}} \\ &= \rho_{X_i,X_j} \end{aligned}$$

Noise addition

- Discussion and summary
  - Noise addition illustrates the tension protection - utility
    - Uncorrelated noise vs. correlated noise

Noise addition

- Discussion and summary
  - Noise addition illustrates the tension protection - utility
    - ▷ Uncorrelated noise vs. correlated noise
  - Connections with differential privacy (Laplacian noise)
    Noise addition with $L(0, \beta)$ and $\beta = \sqrt{(k/2)Var(X)}$
    (variance of $L(\mu, b)$ is $2b^2$
- Multiplicative noise

Noise addition

- Discussion and summary
- Easy to implement, applied independently to each record
- This is for numerical data but
  - PRAM an alternative to categorical data
  - Definition for graphs through random graphs ($G' = G \oplus g$)

PRAM: Post-randomization method

PRAM

PRAM: The Post-Randomization Method

- Description:
  - The scores on some categorical variables for certain records in the original file are changed to a different score.
    - ▷ according to a Markov matrix
- Properties:
  - The Markov approach makes PRAM very general: it encompasses noise addition, data suppression and data recoding.
  - PRAM information loss and disclosure risk largely depend on the choice of the Markov matrix.

PRAM

- Definitions
  - $C = \{c_1, \ldots, c_c\}$ set of categories,
  - P transition matrix on C
    - $P : C \times C \to [0, 1]$ (probabilities)
    - $\sum_{c_j \in C} P(c_i, c_j) = 1$
      The values are positive and rows add to one.

- Procedure
  - Construct X' from X replacing each c_i in X by a c_j with probability $P(c_i, c_j)$.

- Formally, the matrix of probabilities can be seen as a matrix of conditional probabilities.

$$P(c_i, c_j) = P(X' = c_j | X = c_i).$$

PRAM

- Discussion and summary
  - Different matrices, different risk and information loss/utility
  - Research focusing on the matrices

PRAM

- Discussion and summary
  - Invariant PRAM. Frequencies in X and X' the same.
    - $T_X = (T_X(c_1) \dots T_X(c_c))$,
      vector of frequencies of categories in C in the original file X,
    - Then, define P s.t. $\sum_{i=1}^{c} T_X(c_i) p_{ik} = T_X(c_k)$ for all k.
  - Definition
    - If c_k is the category with smaller frequency
      (i.e., that $T_X(c_k) \leq T_X(c_i)$ for all i),
    - and given a parameter θ such that $0 < \theta < 1$,
    - then, define $p_{ij} = P(c_i, c_j)$ as follows:

$$p_{ij} = \begin{cases} 1 - \frac{\theta T_X(c_k)}{T_X(c_i)} & \text{if } i = j \\ \frac{\theta T_X(c_k)}{(c-1) T_X(c_i)} & \text{if } i \neq j \end{cases}$$

PRAM

- Discussion and summary
  - Define the matrix to assign the higher exchange probabilities to the categories with less frequency.
    - They are the ones that have a larger probability of being unique and to make unique the records. This is to increase confusion and reduce the risk of reidentification for the records with these categories.
  - PRAM and transparency
    - To avoid intersection attack, $B_j(x)$ the whole dataset, not null probability for all categories
  - PRAM and differential privacy
    - Randomized response

De-noising data: Lossy compression

Lossy compression

Lossy Compression:

- Description:
  1. The idea is to regard a numerical microdata file as an image
     - records being rows
     - variables being columns
     - values being pixels
  2. Lossy compression (e.g. JPEG) is used on the image
     - Depending on the lossy compression algorithm used, appropriate mappings between variable ranges and color scales will be needed
  3. The compressed image is interpreted as a masked microdata file.

- Example: Description of Lossy Compression using JPEG 80% for a file with 8 records and 8 variables.

Masking Methods for Microdata: Perturbative methods

Rounding:

- Example (for a continuous variable V)
  - Take rounding points as multiples of a base value b:
    - $p_i = b \cdot i$ for $i = 1, \cdots, r$
  - Define the set of attraction for each rounding point:
    - for p_i for $i = 2, \cdots, r-1$, as the interval $[p_i - b/2, p_i + b/2)$,
    - for p_1 and p_r, respectively, the sets of attraction are $[0, p_1 + b/2)$ and $[p_r - b/2, V_{max}]$, where V_{max} is the largest possible value for variable V.
  - An original value v of V is replaced with the rounding point corresponding to the set of attraction where v lies.

Nonperturbative Methods

Masking Methods for Microdata: Nonperturbative methods

Sampling:

- Description
  - Instead of publishing $\mathbf{V} : \mathbf{O} \rightarrow D(V_1) \times D(V_2) \times \cdots \times D(V_m)$
  - Publish: $\mathbf{V}' : \mathbf{S} \rightarrow D(V_1) \times D(V_2) \times \cdots \times D(V_m)$
  - where:
    - $\mathbf{S} \subset \mathbf{O}$ is a sample of the original set of records
    - $\mathbf{V}'$ stands for the original function $\mathbf{V}$ restricted to $\mathbf{S}$.

- Properties:
  - Suitable for categorical microdata
  - Its adequacy for continuous microdata is less clear

- Example: Description of a real-world application of sampling.

Masking Methods for Microdata: Nonperturbative methods

Global recoding:

- Procedure for categorical variables
  - Take a categorical variable V_i
  - Several categories are combined to form new categories
  - A new V_i' with $|D(V_i')| < |D(V_i)|$ where $|\cdot|$ is the cardinality operator.
- Procedure for continuous variables
  - Take a continuous variable V_i
  - Discretize the $D(V_i')$
  - V_i' which is a discretized version of V_i

Masking Methods for Microdata: Nonperturbative methods

Global recoding:

- Properties:
  - More appropriate for categorical microdata
  - High information loss for numerical variables

- Example:
  - consider a record with "Marital status = Widow/er" and "Age = 17"
  - global recoding applied to "Marital status" to create a broader category: "Widow/er or divorced"
  - then, the probability of the above record being unique would diminish

Masking Methods for Microdata: Nonperturbative methods

Top and bottom coding:

- Description
  - A special case of global recoding which can be used on variables that can be ranked
  - Top coding: Top values (above a certain threshold) are lumped together to form a new category
  - Bottom coding: Bottom values (below a certain threshold) are lumped together to form a new category

- Properties:
  - As for global recoding

Masking Methods for Microdata: Nonperturbative methods

Local suppression:

- Description
  - Certain values of individual variables are suppressed
    - ▷ to increase the set of records agreeing on a combination of values

- Properties:
  - Oriented to categorical variables.
  - Methods to combine local suppression and global recoding implemented in μ-Argus SDC package (Hundepool et al. 1998, De Waal and Willenborg 1995)

Masking Methods for Microdata: Nonperturbative methods

Generalization for k-anonymity: Mondrian:

begin

  if not(partitionable(X)) **then**

    return $\{\gamma(x) = \{x \to summary(\mathsf{X})\} | x \in X\}$

  end

  else

    V_i = select variable from X

    i_0 = select a value from domain of V_i in X

    $lhs = \{x \in X | V_i(x) < i_0\}$

    $rhs = \{x \in X | V_i(x) > i_0\}$

    Distribute records in $\{x \in X | V_i(x) = i_0\}$ between lhs and rhs

    return Mondrian(lhs, k) $\cup$ Mondrian(rhs, k)

  end

end

Synthetic data generators

Synthetic Data Generators

Synthetic Data Generators:

$\rightarrow$ seldom pay attention to disclosure risk.

"Since released microdata are synthetic, no real re-identification is possible".

However, unrealistic assumption, if synthetic data generation is performed on the quasi-identifier attributes. Re-identification can indeed happen if a snooper is able to link an external identified data source with some record in the released dataset using the quasi-identifier attributes: coming up with a correct pair (identifier, confidential attributes) is indeed a re-identification.

Synthetic Data Generators: The IPSO family

IPSO-A:

- X and Y two sets of attributes
- X: confidential outcome attributes
- Y: quasi-identifier attributes.
- Then, X are taken as independent and Y as dependent attributes.
- A multiple regression of Y on X is computed and fitted Y'_A attributes are computed. Finally, attributes X and Y'_A are released by IPSO-A in place of X and Y.

In the above setting, conditional on the specific confidential attributes x_i, the quasi-identifier attributes Y_i are assumed to follow a multivariate normal distribution with covariance matrix $\Sigma = \{\sigma_{jk}\}$ and a mean vector $x_i B$, where B is the matrix of regression coefficients.

Synthetic Data Generators: The IPSO family

IPSO B and C:

Let $\hat{B}$ and $\hat{\Sigma}$ be the maximum likelihood estimates of B and Σ derived from the complete dataset (y, x). If a user fits a multiple regression model to (y'_A, x), she will get estimates $\hat{B}_A$ and $\hat{\Sigma}_A$ which, in general, are different from the estimates $\hat{B}$ and $\hat{\Sigma}$ obtained when fitting the model to the original data (y, x).

IPSO-B: Modifies y'_A into y'_B in such a way that the estimate $\hat{B}_B$ obtained by multiple linear regression from (y'_B, x) satisfies $\hat{B}_B = \hat{B}$.

IPSO-C: A more ambitious goal is to come up with a data matrix y'_C such that, when a multivariate multiple regression model is fitted to (y'_C, x), both sufficient statistics $\hat{B}$ and $\hat{\Sigma}$ obtained on the original data (y, x) are preserved.

Synthetic Data Generators: The IPSO family

Experiments for IPSO-A,B,C:

- EIA dataset (4092 records, 15 attributes); Variables used:

Quasi-identifier in external $\mathbf{A}$	Quasi-identifier in released $\mathbf{B}$
$v1$	$v1_A$
$v1, v7, v8$	$v1_A, v7_A, v8_A$
$v1, v2, v7, v8, v9$	$v1_A, v2_A, v7_A, v8_A, v9_A$

- Results:

DBRL1	DBRL2	DBRLM-COV0	DBRLM-COV	KDBRL	PRL
14	9	9	9	14	8
16	15	18	9	16	16
65	121	3206	143	63	159
14	9	9	9	14	8
17	15	18	8	17	16
65	120	3194	135	62	159
11	11	11	11	11	10
6	6	14	8	6	5
53	53	773	46	54	93

Information Loss Measures

Information Loss Measures

Information Loss: information loss depends on the data uses to be supported by the masked data.

- Let X be the original data set on the domain D, and let X' be a protected version of the same data set. Then, for a given data analysis that returns results in a certain domain D' (i.e., $f : D \to D'$), the information loss of f for data sets X and X' is defined by

$$IL_f(X, X') = divergence(f(X), f(X')),$$

where $divergence$ is a way to compare two elements of D'.

Information Loss Measures

Information Loss:

- X, X', D as above, $f : D \to D'$), the information loss of f for data sets X and X' is defined by

$$IL_f(X, X') = divergence(f(X), f(X')),$$

  where $divergence$ is a way to compare two elements of D'.

Reasonable to require:

- $divergence(X, X) = 0$ for all $X \in D$
- $divergence(X, Y) \geq 0$ for all $X, Y \in D'$
- $divergence(X, Y) = divergence(Y, X)$

Information Loss Measures

Information Loss:

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- $divergence(X, Y) = divergence(Y, X)$
  $\to$ asymetric divergence when e.g. to avoid false positives malfunctioning sensor causes huge damage, undetection no.

Information Loss Measures

Information Loss:

- Generic information loss measures
- Specific information loss measures

Information Loss Measures

Information Loss: Generic information loss measures

- A microdata set is analytically valid (Winkler, 1998):
  1. Means and covariances on a small set of subdomains
  2. Marginal values for a few tabulations of the data
  3. At least one distributional characteristic
- A microdata file is analytically interesting if six variables on important subdomains are provided that can be validly analyzed.

Information Loss Measures

- Complementary ways to assess the preservation of the structure of the original data set:
  1. Compare the data in the original and the masked data sets
     - The more similar the SDC method to the identity function, the less impact
  2. Compare some statistics computed on the original and the masked data sets
     - Little information loss should translate to little differences between the statistics
  3. Analyze the behavior of the particular SDC method used

Information Loss Measures

Generic Information loss measures:

- Continuous Data
- Categorical Data

Information Loss Measures: Continuous Data

Characterization of the information in the dataset

- Assume a microdata set X (X' be the masked microdata set) where:
  - n individuals (records) $I_1, I_2, \cdots, I_n$
  - p continuous variables $Z_1, Z_2, \cdots, Z_p$
- The following tools are useful to characterize the information contained in the data set:
  - Covariance matrices V (on X) and V' (on X')
  - Correlation matrices R, R'
  - Correlation matrices RF, RF' between variables and PCA factors $PC_1, \cdots, PC_p$
  - Commonality vectors C, C' between variables and the first principal component (Commonality: the percent of each variable that is explained by PC_1 (or PCi))
  - Factor score coefficient matrices F and F'
    (factors that should multiply each variable in X to obtain its projection on each principal component)

Information Loss Measures: Continuous Data

Matrix divergence

1. Mean square error:

   Sum of squared componentwise differences between pairs of matrices, divided by the number of cells
2. Mean absolute error:

   Sum of absolute componentwise differences between pairs of matrices, divided by the number of cells
3. Mean variation:

   Sum of absolute percent variation of components in the matrix computed on masked data with respect to components in the matrix computed on original data, divided by the number of cells.

Information Loss Measures: Continuous Data

	Mean square error	Mean abs. error	Mean variation
$X - X'$	$\frac{\sum_{j=1}^{p}\sum_{i=1}^{n}(x_{ij}-x'_{ij})^2}{np}$	$\frac{\sum_{j=1}^{p}\sum_{i=1}^{n}\lvert x_{ij}-x'_{ij}\rvert}{np}$	$\frac{\sum_{j=1}^{p}\sum_{i=1}^{n}\frac{\lvert x_{ij}-x'_{ij}\rvert}{\lvert x_{ij}\rvert}}{np}$
$V - V'$	$\frac{\sum_{j=1}^{p}\sum_{1\le i\le j}(v_{ij}-v'_{ij})^2}{\frac{p(p+1)}{2}}$	$\frac{\sum_{j=1}^{p}\sum_{1\le i\le j}\lvert v_{ij}-v'_{ij}\rvert}{\frac{p(p+1)}{2}}$	$\frac{\sum_{j=1}^{p}\sum_{1\le i\le j}\frac{\lvert v_{ij}-v'_{ij}\rvert}{\lvert v_{ij}\rvert}}{\frac{p(p+1)}{2}}$
$R - R'$	$\frac{\sum_{j=1}^{p}\sum_{1\le i<j}(r_{ij}-r'_{ij})^2}{\frac{p(p-1)}{2}}$	$\frac{\sum_{j=1}^{p}\sum_{1\le i<j}\lvert r_{ij}-r'_{ij}\rvert}{\frac{p(p-1)}{2}}$	$\frac{\sum_{j=1}^{p}\sum_{1\le i<j}\frac{\lvert r_{ij}-r'_{ij}\rvert}{\lvert r_{ij}\rvert}}{\frac{p(p-1)}{2}}$
$RF - RF'$	$\frac{\sum_{j=1}^{p} w_j \sum_{i=1}^{p}(rf_{ij}-rf'_{ij})^2}{p^2}$	$\frac{\sum_{j=1}^{p} w_j \sum_{i=1}^{p}\lvert rf_{ij}-rf'_{ij}\rvert}{p^2}$	$\frac{\sum_{j=1}^{p} w_j \sum_{i=1}^{p}\frac{\lvert rf_{ij}-rf'_{ij}\rvert}{\lvert rf_{ij}\rvert}}{p^2}$
$C - C'$	$\frac{\sum_{i=1}^{p}(c_i-c'_i)^2}{p}$	$\frac{\sum_{i=1}^{p}\lvert c_i-c'_i\rvert}{p}$	$\frac{\sum_{i=1}^{p}\frac{\lvert c_i-c'_i\rvert}{\lvert c_i\rvert}}{p}$
$F - F'$	$\frac{\sum_{j=1}^{p} w_j \sum_{i=1}^{p}(f_{ij}-f'_{ij})^2}{p^2}$	$\frac{\sum_{j=1}^{p} w_j \sum_{i=1}^{p}\lvert f_{ij}-f'_{ij}\rvert}{p^2}$	$\frac{\sum_{j=1}^{p} w_j \sum_{i=1}^{p}\frac{\lvert f_{ij}-f'_{ij}\rvert}{\lvert f_{ij}\rvert}}{p^2}$

Information Loss Measures: Categorical Data

Alternative definitions of information loss measures:

- Direct comparison of categorical values
- Comparison of contingency tables
- Entropy-based measures

Information Loss Measures: Categorical Data

Direct comparison of categorical values:

Comparison of matrices X and X' requires the definition of a distance.
For nominal variables:

$$d_V(c, c') = \begin{cases} 0 & \text{when } c = c' \\ 1 & \text{when } c \neq c' \end{cases}$$

For ordinal variables ($\leq_V$ be the total order):

$$d_V(c, c') = \frac{|\{c'' : min(c, c') \leq_V c'' \leq_V max(c, c')\}|}{|D(V)|}$$

Information Loss Measures: Categorical Data

Comparison of contingency tables:

F original data set, G masked data set

t-dimensional contingency tables ($t \leq K$)

$x^{file}_{subscripts}$: entry of the contingency table of file at position subscripts

$$CTBIL(F, G; W, K) = \sum_{\substack{\{V_{j1} \cdots V_{jt}\} \subseteq W \\ |\{V_{j1} \cdots V_{jt}\}| \leq K}} \sum_{i_1 \cdots i_t} |x^F_{i_1 \cdots i_t} - x^G_{i_1 \cdots i_t}|$$

Information Loss Measures: Categorical Data

Entropy-based measures:

- Entropy is an information-theoretic measure, but can be used in SDC if the masking process is modeled as the noise that would be added to the original data set in the event of it being transmitted over a noisy channel.
- For PRAM: Assume $P_{V,V'} = \{p(V' = j|V = i)\}$ the PRAM matrix

Procedure:

- The conditional uncertainty of V given that $V' = j$ is:

$$H(V|V' = j) = -\sum_{i=1}^{n} p(V = i|V' = j) log\ p(V = i|V' = j)$$

- Entropy-based information loss measure (EBIL):

$$EBIL(P_{V,V'}, G) = \sum_{r \in G} H(V|V' = j_r)$$

where j_r is the value taken by V' in record r. Note $H(V|V' = j_r)$ do not depend on the value in V

Information Loss Measures: Categorical Data

An alternative Information Loss Measure:

- EBIL a function of the masked data set but does not depend on the original data set.
- Assume that, in a household survey file, variable V contains the town where the household is located. Now consider that V is masked into a new variable V' where the town has been replaced by the state. Locations like "New York City" and "Albany" will be recoded into "NY". Living in Albany is more specific and identifying (in the sense of being less anonymous) than living in New York City. The information loss measure should somehow reflect that there is more information loss when a household in "Albany" becomes a household in "New York State" than when a household in "New York City" becomes a household in "New York State"
- Note that: $P(V = \text{“}Albany\text{''}|V' = NY) < P(V = \text{“}NewYorkCity\text{''}|V' = NY)$
- According to the USBC American FactFinder, the population of New York State in 2000 was $17,990,455$, the population of New York City was $7,322,564$ and the population of Albany was $101,082$. Thus, the above probabilities are $P(V = \text{“}Albany\text{''}|V' = NY) = 101,082/17,990,455 = 0.05$ and $P(V = \text{“}NewYorkCity\text{''}|V' = NY) = 7,322,564/17,990,455 = 0.407$.

Information Loss Measures: Categorical Data

An alternative Information Loss Measure:

- The smaller the conditional probability $P(V = i|V' = j)$, the larger the inf. loss.
- Information loss as a function of three elements:
  (i) conditional probability; (ii) original category i; (iii) masked category j
- Per-record information loss when $V = i$ is masked as $V' = j$ can be defined as:

$$PRIL(PV, V', i, j) = -logP(V = i|V' = j)$$

- The information loss for the entire data sets F, G is

$$IL(PV, V', F, G) = \sum_{r \in G} PRIL(P_{V,V'}, i_r, j_r)$$

where i_r is the value taken by V in record r of F (similarly, j_r in G)

Information Loss Measures

Specific Information Loss Measure:

- Types of measures
- Do generic measures approximate specific ones?

Information Loss Measures

Specific Information Loss Measure: An example

- data use: clustering cl with parameter c
- divergence: Rand, Jaccard, Adjsted Rand Index, Wallace, Mantaras, ...

$$IL_{Rand,cl}(X, X') = 1 - Rand(cl_c(X), cl_c(X'))$$

$$IL_{Mantaras,cl}(X, X') = Mantaras(cl_c(X), cl_c(X'))$$

Information Loss Measures

Specific Information Loss Measure: Some results:

- Census data set microaggregated (3 vars at a time, different k), k-means with c=15. Cols 2-6: Indices/distance; col 7: averaged probabilistic information loss measure (aPIL); (c) last row is the correlation of the measures and distance with respect to the aPIL.

	Rand	Jaccard	Adjusted Rand	Wallace	Mantaras	aPIL
Mic3vars.k3	0.943	0.454	0.594	0.625	0.416	15.189
Mic3vars.k4	0.943	0.464	0.602	0.633	0.425	19.325
Mic3vars.k5	0.936	0.406	0.542	0.577	0.472	22.724
Mic3vars.k6	0.936	0.408	0.545	0.580	0.473	25.760
Mic3vars.k7	0.929	0.367	0.499	0.537	0.500	28.750
Mic3vars.k8	0.933	0.402	0.538	0.574	0.479	31.185
Mic3vars.k9	0.925	0.359	0.488	0.528	0.513	33.883
Correlation	-0.930	-0.882	-0.887	-0.882	0.931	1.000

Information Loss Measures

Specific Information Loss Measure: Some results:

- Census data set. Correlations same file.

Index / Distance	Correlation (a) all (215 files)	Correlation (b) Microaggregation (162 files)
Rand Index	-0.79281	-0.86099
Jaccard Index	-0.89094	-0.94859
Adjusted Rand Index	-0.91609	-0.96114
Wallace Index	-0.92559	-0.97593
Mantaras Distance	0.91617	0.97216

Information Loss Measures

Specific Information Loss Measure: Some results:

- Census data set. Convergence problems
- Census dataset with additive noise. Fuzzy clustering with $c = 10$. OF for the original file 2851 in the first execution (left), 2829 in the second. d_1 distance between cluster centers, d_2 distance between membership values.

Noise	d_1	d_2	$O.F.$	d_1	d_2	$O.F.$
0.0	3.21	40.73	2826.0	3.93	91.3	2826
0.1	3.21	40.67	2827.0	3.97	91.45	2827
0.2	3.17	40.86	2829.0	3.94	90.89	2829
0.4	0.32	0.92	2859.0	4.07	93.05	2835
0.6	3.28	42.09	2844.0	6.92	113.76	2867
0.8	3.48	43.48	2862.0	4.19	91.53	2862
1.0	3.55	48.87	2886.0	4.37	99.33	2886
1.2	2.24	55.56	2908.0	2.75	68.04	2903
1.4	1.44	18.35	2935.0	4.53	99.53	2918
1.6	2.27	36.83	2978.0	6.98	103.84	2978
1.8	2.71	45.59	3006.0	4.68	99.20	2989
2.0	4.24	96.87	3028.0	2.70	31.17	3013

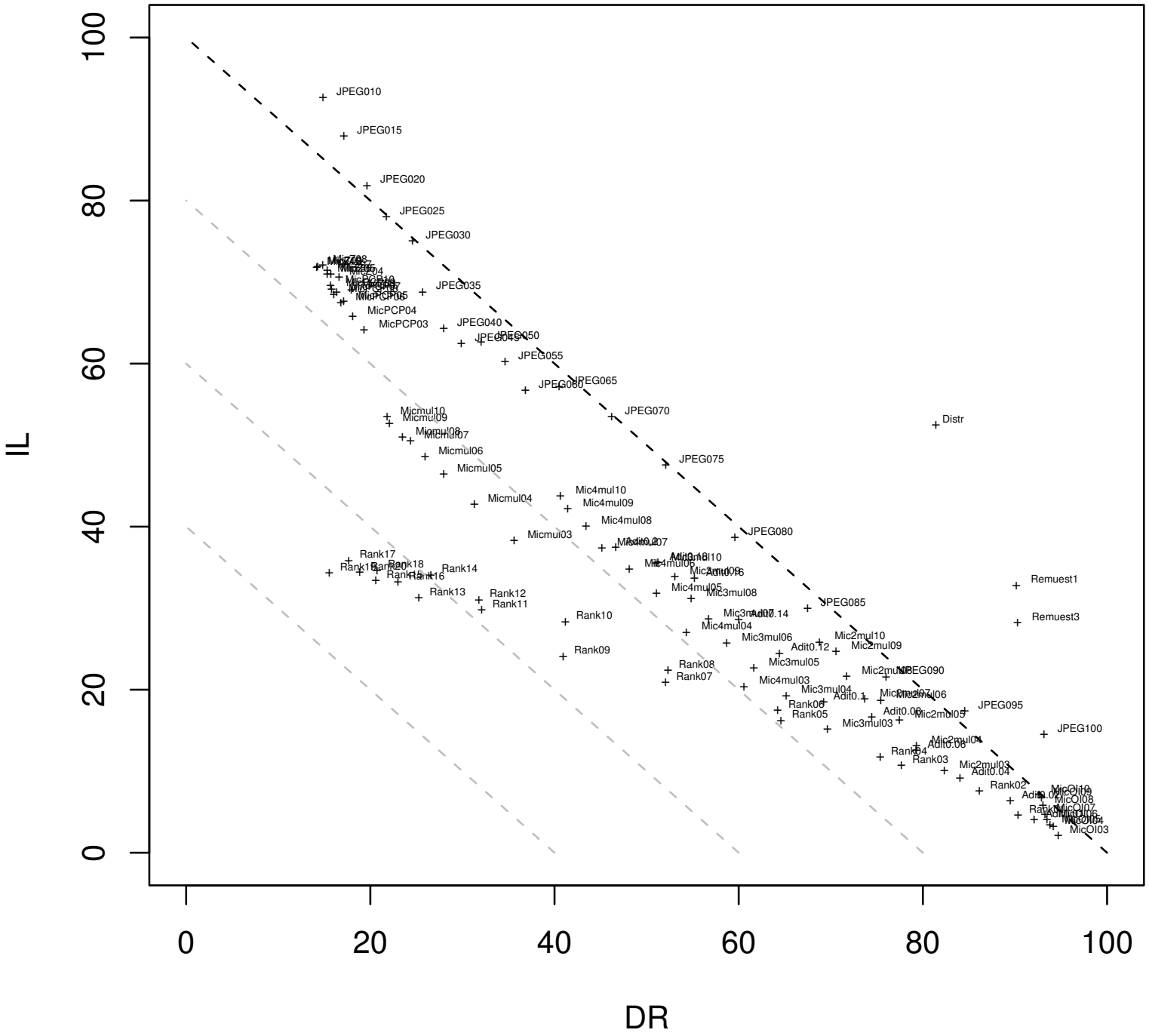
Visualization

Visualization

Trade-off:

- Information loss and disclosure risk are usually in conflict
- R-U maps
  - Graphical representation
- Score
  - R-U maps

Risk/Utility Map



Visualization

Trade-off: Score

$$Score(X, X') = \frac{IL(X, X') + DR(X, X')}{2}$$

Summary

Visualization

Data privacy

- Masking methods
- Information loss
- Disclosure risk
- Visualization