Privacy models and disclosure risk

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1. Disclosure

- 2. Privacy models
- 3. Classification of privacy models and disclosure risk measures
 - Attribute disclosure
 - Identity disclosure (record linkage and worst-case scenario)
- 4. k-Anonymity

Disclosure

Disclosure risk. Disclosure = leakage of information.

- Identity disclosure vs. Attribute disclosure
 - Attribute disclosure: (e.g. learn about Alice's salary)
 - \star Increase knowledge about an attribute of an individual
 - Identity disclosure: (e.g. find Alice in the database)
 Find /identify an individual in a database (a.g. masked)
 - * Find/identify an individual in a database (e.g., masked file)

Within machine learning, some attribute disclosure is expected.

Privacy models

Privacy models



Privacy models: What is a privacy model ?

• To make a program we need to know what we want to protect

Definition:

• A computational definition for privacy

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• To make a program we need to know what we want to protect

Definition:

• A computational definition for privacy

Quite a large number of *computational definitions*, they depend on what to protect.

Privacy models. A computational definition for privacy. Examples.

- **Reidentification privacy.** Avoid finding a record in a database.
- k-Anonymity. A record indistinguishable with k-1 other records.
- Secure multiparty computation. Several parties want to compute a function of their databases, but only sharing the result.
- **Differential privacy.** The output of a query to a database should not depend (much) on whether a record is in the database or not.
- **Result privacy.** We want to avoid some results when an algorithm is applied to a database.
- Integral privacy. Inference on the databases. E.g., changes have been applied to a database.
- Homomorphic encryption. We want to avoid access to raw data and partial computations.

Privacy models. A computational definition for privacy. Publish a DB

- Reidentification privacy. Avoid finding a record in a database.
- **k-Anonymity.** A record indistinguishable with k 1 other records.
- k-Anonymity, I-diversity. *l* possible categories
- Interval disclosure. The value for an attribute is outside an interval computed from the protected value: values different enough.
- **Result privacy.** We want to avoid some results when an algorithm is applied to a database.



Privacy models

Privacy models. A computational definition for privacy. Publish a DB

• Modify DB X to obtain a DB X' compliant with the privacy model.

	Respondent	t City	Ag	e III	ness	
Original DB X:	DRR	Barcelon	a 30) Heart	attack	
	ABD	ABD Barcelon		2 Ca	Cancer	
	COL	COL Barcelona		8 Ca	Cancer	
	GHE	Tarragon	a 62	2 A	AIDS	
	CIO	Tarragon	a 65	5 А	AIDS	
	HYU	Tarragon	a 60) Heart	attack	
Published DB X':						
		City	Age	Illness		
	—	Barcelona	30	Cancer		
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	_	Barcelona	30	Cancer		
	_	Tarragona	60	AIDS		
	—	Tarragona	60	AIDS		
			—			

- Difficulties: naive anonymization does not work
 - \circ (Sweeney, 1997; 2000¹) on USA population
 - * 87.1% (216 /248 million) is likely to be uniquely identified by 5-digit ZIP, gender, date of birth,
 - * 3.7% (9.1 /248 million) is likely to be uniquely identified by 5-digit ZIP, gender, Month and year of birth.
- Difficulties: highly identifiable data and high dimensional data
 - Data from mobile devices:
 - * two positions can make you unique (home and working place)
 - AOL and Netflix cases (search logs and movie ratings)
 - Similar with credit card payments, shopping carts, search logs, ... (i.e., high dimensional data)

¹L. Sweeney, Simple Demographics Often Identify People Uniquely, CMU 2000

- Difficulties: Example 1.
 - Q: sickness influenced by studies & commuting distance?
 - Records: (where students live, what they study, if they got sick)

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 - Q: sickness influenced by studies & commuting distance?
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 - No "personal data",
 - DB = { (Dublin, CS, No), (Dublin, CS, No), (Dublin, CS, Yes), (Maynooth, CS, No), . . . , (Dublin, BA MEDIA STUDIES, No) (Dublin, BA MEDIA STUDIES, Yes), . . . }
 - is this ok ?
 - NO!!:
 - E.g., there is only one student of anthropology living in Enfield. (Enfield, Anthropology, Yes)

- Difficulties (summary)
 Naive anonymization does not work,
 highly identifiable data, high dimensional data
- Examples of successful reidentification attacks
 Sweeney analysis of USA population, data from mobile data, shopping cards, film ratings

Privacy models. A computational definition for privacy. Share a result

• Secure multiparty computation. Several parties want to compute a function of their databases, but only sharing the result.



Privacy models. A computational definition for privacy. Share a result

• Compute

 $f(DB_1, DB_2, DB_3, DB_4)$

without sharing DB_1, DB_2, DB_3, DB_4

• Example: national age mean of hospital-acquired infection patients (hospitals do not want to share the age of their infected patients!)

• Difficulties

- Distributed approach (no trusted-third party) computational cost of solutions
- Protocols only valid for a particular function

Privacy models. A computational definition for privacy. Compute result

- **Differential privacy.** The output of a query to a database should not depend (much) on whether a record is in the database or not.
- Integral privacy. Inference on the databases. E.g., changes have been applied to a database.
- Homomorphic encryption. We want to avoid access to raw data and partial computations.



- Difficulties. Output of a function can be sensitive. Example 2
 - Mean income of admitted to hospital unit (e.g., psychiatric unit)
 Mean salary of participants in Alcoholics Anonymous by town
 Is this ok? NO!!
 - disclosure of a rich person in the database

Privacy models: Summary

- Privacy models: quite a few competing models
 - differential privacy
 - secure multiparty computation
 - k-anonymity
 - k-Anonymity, I-diversity
 - computational anonymity
 - reidentification (record linkage)
 - uniqueness
 - result privacy
 - interval disclosure
 - integral privacy

Privacy models: Summary

- Privacy models: quite a few competing models
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 - reidentification (record linkage)
 - uniqueness
 - result privacy
 - interval disclosure
 - integral privacy
- ... and combined:

 \circ secure multiparty computation + differential privacy

Classification of privacy models and disclosure risk measures

Disclosure risk.

- Boolean vs. quantitative privacy models
 - Boolean: Disclosure either takes place or not. Check whether the definition holds or not. Includes definitions based on a threshold.
 - Quantitative: Disclosure is a matter of degree that can be quantified. Some risk is permitted.
- Implication when selecting a method
 - minimize information loss (max. utility) vs.
 multiobjetive optimization

Disclosure risk.

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- Boolean vs. quantitative measures/models

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Classification of privacy models (and measures)

Attribute disclosure Identity disclosure

Bool	lean
DUUU	Call

Quantitative

Differential privacy Result privacy Secure multipar	k–Anonymity ty computation	
Interval disclosure	Re-identification (record linkage) Uniqueness	

Boolean definitions of risk.

- k-Anonymity (Boolean definition / identity disclosure)
- Secure multiparty computation (Boolean / identity and attribute disclosure)
- Result privacy (Boolean definition / attribute disclosure)
- Differential privacy (Boolean definition / attribute disclosure)

Quantitative measures of risk. alternative measures.

- Re-identification (for identity disclosure). Different ways to evaluate re-identification by means of record linkage.
- Uniqueness (for identity disclosure).
- Interval disclosure (for attribute disclosure). Several definitions for different types of attributes.

Classification of privacy models (and measures)

Attribute disclosure Identity disclosure

Boolean	Differential privacy Result privacy Secure multipar	k–Anonymity ty computation	
Quantitative	Interval disclosure	Re-identification (record linkage) Uniqueness	

Other privacy models

- Other models combining features: I-diversity, secure multiparty computation ensuring differential privacy
- Alternative but related models: k-confusion, k-concealment

Attribute disclosure (database protection)

- Algorithm Rank-based interval disclosure: rid(X, V, V', x, p)
 - Input: X: Original file; V: Original attribute; V': Masked attribute;
 x: record; p: percentage
 - **Output:** Attribute disclosure for attribute V' of record x
 - $\circ R(V) := \mathsf{Rank} \mathsf{ data} \mathsf{ for attribute} V'$
 - $\circ \ i := \text{position of } V'(x) \text{ in } R(V)$
 - $\circ w := p \cdot |X|/2/100$ (width of the interval)
 - $I(x) = [R[\min(i w, 0)], R[\max(i + w, |X| 1)]]$ (definition of the interval for record x)
 - $\circ \ rid := V(x) \in I(x)$
 - **Return** *rid*

- Algorithm Standard deviation-based interval disclosure: sdid(X, V, V', x, p)
 - Input X: Original file; V: Original attribute; V': Masked attribute;
 x: record; p: percentage
 - $\circ~\mathbf{Output}$ Attribute disclosure for attribute V' of record x
 - $\circ \ sd(V) := standard deviation of V$
 - $\circ \ sdid := |V(x) V'(x)| \le p \cdot sd(V)/100$
 - **Return** *sdid*

Uniqueness (database protection)

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 - Suitable for sampling ($\rho(X)$ is a subset of X).
 - For masked data, the same combination will not appear.

Uniqueness

Measures for identity disclosure: Uniqueness (categorical data/sampling)

• File-level uniqueness. It is defined as the probability that a sample unique (SU) is a population unique (PU). The following expression has been used:

$$P(PU|SU) = \frac{P(PU, SU)}{P(SU)} = \frac{\sum_{j} I(F_j = 1, f_j = 1)}{\sum_{j} I(f_j = 1)}$$

where $j = 1, \ldots, J$ denotes possible values in the sample, F_j is the number of individuals in the population with key value j (frequency of j in the population), f_j is the same frequency for the sample and I stands for the cardinality of the selection.

• **Record-level risk uniqueness.** It is defined as the probability that a particular sample record is re-identified (recognized as corresponding to a particular individual in the population).
Identity disclosure (database protection)

- Privacy from re-identification. Identity disclosure. Scenario:
 - \circ A: File with the protected data set
 - \circ B: File with the data from the intruder (subset of original X)



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How to establish the correct links between the two files? Record linkage algorithms (used in e.g. database integration)

- Privacy from re-identification. Identity disclosure.
 - \circ A: File with the protected data set
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How to establish the correct links between the two files? Record linkage algorithms (used in e.g. database integration)

- Two main types.
 - Distance-based record linkage
 - Probabilistic record linkage

- Distance-based record linkage: d(a, b) with $a \in A$ and $b \in B$.
 - Assign to the record at a minimum distance, ideally an intruder wants for a record *i*: $d(a_i, b_j) \ge d(a_i, b_i)$ for all *j* but due to masking we expect this does not happen



- Algorithm Distance-based record linkage
 - \circ **Input** A: file; B: file
 - \circ **Output** *LP*: linked pairs; *NP*: non-linked pairs
 - \circ For $a \in A$
 - $\circ \qquad \mathsf{b'} = \arg \min_{b \in B} d(a, b)$
 - $\circ \qquad LP = LP \cup (a, b')$
 - for $b \in B$ such that $b \neq b'$
 - $\circ \qquad NP := NP \cup (a, b)$
 - end for
 - end for
 - \circ **Return** (*LP*, *NP*)

- Probabilistic record linkage: d(a, b) with $a \in A$ and $b \in B$.
 - $\circ\,$ Classification of pairs of records (a,b) in 3 classes Linked pair, non-linked, clerical pair
 - How?
 - \star For each pair (a,b), an index is computed using the conditional probabilities
 - $\ P(coincidence|Matching)$: coincidence between both records when there is matching
 - P(coincidence|Unmatching): coincidence between both records when there is no matching
 - ***** Classification using thresholds

- Probabilistic record linkage: d(a, b) with $a \in A$ and $b \in B$.
 - $\circ\,$ Computation of

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P(coincidence|Matching) and P(coincidence|Unmatching):
* Using EM algorithm
• Computation of thresholds

* From the probabilities of false positive/negative

P(Linkedpair|Unmatching)P(Nonlinkedpair|Matching)

- Flexible scenario for identity disclosure
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• But also,

- $\star B$ with a schema different to the one of A (different attributes)
- * Other scenarios. E.g., synthetic data
- * Other type of data: graph data (reidentifying people in a social network)

- **Privacy from re-identification**. Worst-case scenario (maximum knowledge) to give upper bounds of risk:
 - transparency attacks (information on how data has been protected)
 - largest data set (original data)
 - best re-identification method (best record linkage/best parameters)



- Privacy from re-identification. Worst-case scenario.
 - ML for distance-based record linkage parameters. (A and B aligned)
 ★ Goal: as many correct reidentifications as possible:
 - for each record $i: d(a_i, b_j) \ge d(a_i, b_i)$ for all j



 \blacksquare $d(a_i, b_j)$ as average/sum of attribute/variable distances

 $\mathbb{C}_p(diff_1(a_i, b_j), \ldots, diff_n(a_i, b_j))$

- Privacy from re-identification. Worst-case scenario.
 - ML for distance-based record linkage parameters. (A and B aligned)
 ★ Goal: as many correct reidentifications as possible. But, if error for a_i: K_i = 1 and d(a_i, b_j)+CK_i ≥ d(a_i, b_i) for all j where d is an aggregated distance d(a, b) = C_p(diff₁,..., diff_n):
 ★ Formally,

 $\mathbb{C}_p(diff_1(a_i, b_j), \dots, diff_n(a_i, b_j)) + CK_i \ge \mathbb{C}_p(diff_1(a_i, b_i), \dots, diff_n(a_i, b_i))$

- Privacy from re-identification. Worst-case scenario.
 - \circ ML for distance-based record linkage parameters. (A and B aligned)
 - Goal: as many correct reidentifications as possible.
 - Minimize K_i : minimize the number of records a_i that fail
- Formalization:

$$Minimize\sum_{i=1}^{N} K_i$$

Subject to :

$$\mathbb{C}_{p}(diff_{1}(a_{i}, b_{j}), \dots, diff_{n}(a_{i}, b_{j})) - \\ - \mathbb{C}_{p}(diff_{1}(a_{i}, b_{i}), \dots, diff_{n}(a_{i}, b_{i})) + CK_{i} > 0$$
$$K_{i} \in \{0, 1\}$$
Additional constraints according to \mathbb{C}

- Privacy from re-identification. Worst-case scenario.
 - \circ ML for distance-based record linkage parameters. (A and B aligned)
 - The case of the weighted mean $(\mathbb{C} = WM)/W$ eighted Euclidean • Formalization:

$$d^{2}(a,b) = WM_{p}(diff_{1}(a,b),\ldots,diff_{n}(a,b))$$

with arbitrary vector $p = (p_1, \dots, p_n)$ and $diff_i(a, b) = ((a_i - \overline{a}_i)/\sigma(a_i) - (b_i - \overline{b}_i)/\sigma(b_i))^2$

- Privacy from re-identification. Worst-case scenario.
 - \circ ML for distance-based record linkage parameters. (A and B aligned)
 - The case of the weighted mean ($\mathbb{C} = WM$)
 - Formalization:

$$\begin{split} Minimize \ &\sum_{i=1}^{N} K_i \\ Subject \ to: \ &WM_p(diff_1(a_i, b_j), \dots, diff_n(a_i, b_j)) - \\ &- WM_p(diff_1(a_i, b_i), \dots, diff_n(a_i, b_i)) + C \ K_i > 0 \\ &K_i \in \{0, 1\} \\ &\sum_{i=1}^{n} p_i = 1 \\ &p_i \ge 0 \end{split}$$

- Privacy from re-identification. Worst-case scenario.
 - \circ ML for DBRL parameters: Distances considered $\mathbb C$
 - ★ Weighted mean.
 - Weights: importance to the attributes
 - Parameter: weighting vector n = # attributes

- Privacy from re-identification. Worst-case scenario.
 - \circ ML for DBRL parameters: Distances considered $\mathbb C$
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 - Parameter: weighting vector n = # attributes
 - * OWA linear combination of order statistics (weighted): Weights: to discard lower or larger distances Parameter: weighting vector n = # attributes
 - * Bilinear form generalization of Mahalanobis distance Weights: interactions between pairs of attributes Parameter: square matrix: $n \times n$ (n = # attributes)
 - * Choquet integral.

Weights: interactions of sets of attributes $(\mu : 2^X \rightarrow [0, 1])$ Parameter: non-additive measure: $2^n - 2$ (n = # attributes) Distances used in record linkage based on aggregation operators

• Graphically



Choquet integral. A fuzzy integral w.r.t. a fuzzy measure (non-additive measure). CI generalizes Lebesgue integral. Interactions.

k-Anonymity (a privacy model)

• Definition. Let $RT(A_1, \ldots, A_n)$ be a table, and QI_{RT} be the quasi-identifier associated with it. RT is said to satisfy k-anonymity if and only if each sequence of values in $RT[QI_{RT}]$ appears with at least k occurrences in $RT[QI_{RT}]$.

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- **Example.** k-anonymous table for k = 2 when the $QI_{RT} = {City,age}$.

Respondent	City	age	illness
ABD	Barcelona	30	Cancer
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GHE	Tarragona	60	AIDS
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 - \circ The definition of $k\mbox{-anonymity}$ makes that algorithms focus on information loss.
 - $\circ\,$ Different levels of k lead to different protections
 - $\circ~k\mbox{-Anonymity through generalization}$ and suppression: NP-Hard problem

Disclosure Risk

k-Anonymity

• Attacks. (I)

Disclosure Risk

k-Anonymity

- Attacks. (I)
 - Homogeneity attack. When all indistinguishable records in a cluster are also indistinguishable with respect to a confidential variable, attribute disclosure can take place.
k-Anonymity

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 - Homogeneity attack. When all indistinguishable records in a cluster are also indistinguishable with respect to a confidential variable, attribute disclosure can take place.

• **Example.**

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k-Anonymity

- Attacks. (II)
 - External knowledge attack. In this case, some information about an individual is used to deduce information of the same or another individual.
 - **Example.** If we are HYU, we can deduce that CIO has AIDS (without reidentification).

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k-Anonymity: Extensions (I)

• *p*-Sensitive *k*-anonymity. (Truta, Vinay, 2006)

A data set is said to satisfy *p*-sensitive *k*-anonymity for k > 1 and $p \le k$ if it satisfies *k*-anonymity and, for each group of records with the same combination of values for quasi-identifiers, the number of distinct values for each confidential value is at least p (within the same group).

k-Anonymity: Extensions (II)

- *l*-**Diversity.** (Machanavajjhala et al. 2006)
 - It forces *l* different categories in each set. However, in this case, categories should have to be *well-represented*. Different meanings have been given to what *well-represented* means.

k-Anonymity: Extensions (III)

• *t*-closeness. (Li, Li, Venkatasubramanian, 2007)

The distribution of the attribute in any k-anonymous subset of the database is similar to the one of the full database. Similarity is defined in terms of the distance between the two distributions and such distance should be below a given threshold t.

Low threshold makes the utility of the data doubtful: large information loss.